Role Model Based Mechanism For Norm Emergence In Artificial Agent Societies

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Abstract. In this paper we propose a mechanism for norm emergence based on role models. The mechanism uses the concept of normative advice whereby the role models provide advice to the follower agents. Our mechanism is built using two layers of networks, the social link layer and the leadership layer. The social link network represents how agents are connected to each other. The leadership network represents the network that is formed based on the role played by each agent on the social link network. The two kinds of roles are leaders and followers. We present our findings on how norms emerge on the leadership network when the topology of the social link network changes. The three kinds of social link networks that we have experimented with are fully connected networks, random networks and scale-free networks.

1 Introduction

Norms are a widely observed mechanism for enforcing discipline and prescribing uniform behaviour in human societies. Norms specify the way the members of a society should behave and help societies to improve co-operation and collaboration among their members [1]. Some examples of norms in modern societies include the exchange of gifts at Christmas, tipping in restaurants and dinner table etiquette.

Norms have been so much a part of different cultures, it is not surprising that it is an active area of research in a variety of fields including Sociology, Economics, Biology and Computer Science. However, norms have been of interest to multi-agent researchers only for a decade now. Norms are of interest to the MAS researchers as software agents tend to deviate from these norms due to their autonomy. So, the study of norms has become crucial to MAS researchers so that they can build robust multi-agent systems that comply to norms and also systems that evolve and adapt norms dynamically.

Our objective in this paper is to propose a mechanism based on role models for norm emergence using the concept of oblique norm transmission in artificial agent societies. We will demonstrate that our mechanism results in complete norm emergence (100% norm emergence) by using it on top of three kinds of network topologies.

2 Background

In this section we describe different types of norms and the treatment of norms in multi-agent systems. We also describe the work related to norm emergence and different kinds of network topologies.

2.1 Types of norms

Due to multi-disciplinary interest in norms, several definitions for norms exist. Habermas [2], a renowned sociologist, identified norm regulated actions as one of the four action patterns in human behaviour. A norm to him means fulfilling a generalized expectation of behaviour, which is a widely accepted definition for social norms. Researchers have divided norms into different categories. Tuomela [3] has categorized norms into the following categories.

- r-norms (rule norms)
- s-norms (social norms)
- m-norms (moral norms)
- p-norms (prudential norms)

Rule norms are imposed by an authority based on an agreement between the members (e.g. one has to pay taxes). Social norms apply to large groups such as a whole society (e.g. one should not litter). Moral norms appeal to one's conscience (e.g. one should not steal or accept bribe). Prudential norms are based on rationality (e.g one ought to maximize one's expected utility). When members of a society violate the societal norms, they may be punished.

Many social scientists have studied why norms are adhered to. Some of the reasons for norm adherence include:

- fear of authority or power
- rational appeal of the norms
- emotions such as shame, guilt and embarrassment that arise because of nonadherence.
- willingness to follow the crowd

Elster [4] categorizes norms into consumption norms (e.g. manners of dress), behaviour norms (e.g. the norm against cannibalism), norms of reciprocity (e.g. gift-giving norms), norms of cooperation (e.g. voting and tax compliance) etc.

2.2 Normative multi-agent systems

Research on norms in multi-agent systems is fairly recent [5–7]. Norms in multi-agent systems are treated as constraints on behaviour, goals to be achieved or as obligations [8]. There are two main research branches in normative multi-agent systems. The first branch focuses on normative system architectures, norm representations and norm adherence and the associated punitive or incentive measures. The second branch of research is related to emergence of norms.

Lopez et al. [9] have designed an architecture for normative BDI agents and Boella et al. [10] have proposed a distributed architecture for normative agents. Some researchers are working on using deontic logic to define and represent norms [11, 10]. Several researchers have worked on mechanisms for norm compliance and enforcement [12–14]. A recent development is the research on emotion based mechanism for norm enforcement by Fix et al. [15]. Conte and Castelfranchi [16] have worked on an integrated view of norms, from the perspectives of Sociology and Economics. Their views are similar to that of Elster [4].

2.3 Related work on emergence of norms

The second branch of research on norms focuses on two main issues. The first issue is on norm propagation within a particular society. According to Boyd and Richerson [17], there are three ways by which a social norm can be propagated from one member of the society to another. They are

- Vertical transmission (from parents to offspring)
- Oblique transmission (from a leader of a society to the followers)
- Horizontal transmission (from peer to peer interactions)

Norm propagation is achieved by spreading and internalization of norms [6, 18]. Boman and Verhagen [6, 19, 18] have used the concept of normative advice (advice from the leader of a society) as one of the mechanisms for spreading and internalizing norms in an agent society. The concept of normative advice in their context is based on an assumption that the norm has been accepted by the top level enforcer, the Normative Advisor, and the norm does not change. But, this context cannot be assumed for scenarios where norms are being formed (when the norms undergo changes).

So, the second issue that has received less attention is the emergence of norms. However, there is abundant literature in the area of sociology on why norms are accepted in agent societies and how they might be passed on. Karl-Dieter Opp [20] has proposed a theory of norm emergence from a sociology perspective. Epstein [21] has proposed a model of emergence based on the argument that the norms reduce individual computations.

The treatment of norms has been mostly in the context of an agent society where the agents interact with all the other agents in the society [19, 21, 22]. Few researchers have considered the actual topologies of the social network for norm emergence [23, 24]. We consider that social networks are of importance to the emergence of norms as they provide the topology and the infrastructure on which the norms can be exchanged. We are inspired by previous works on the spreading of ideas (opinion dynamics [25]) and diseases [26] over different network topologies.

Social networks are important for norm emergence because in the real world, people are not related to each other by chance. They are related to each other through the social groups that they are in, such as the work group, church group, ethnic group and the hobby group. Information tends to percolate among

the members of the group through interactions. Also, people seek advice from a close group of friends and hence information gets transmitted between the members of the social network. Therefore, it is important to test our mechanism for norm emergence on top of social networks, a topic which is receiving attention among multi-agent researchers recently [24]. Network topologies have also been explored by multi-agent system researchers in other contexts such as reputation management [27, 28].

2.4 Social network topologies

In this section we describe three network topologies that we have considered for experimenting with norm emergence.

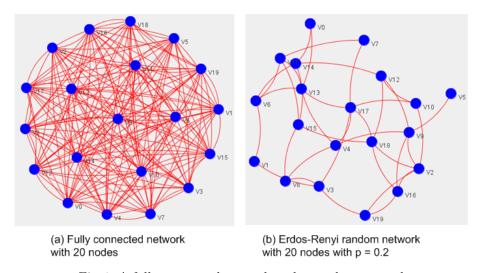


Fig. 1: A fully connected network and a random network

Fully connected network: In the fully connected network topology, each agent in the society is connected to all the agents in a given society (shown in Figure 1(a)). Many multi-agent researchers have done experiments with this topology. Most of their experiments involve interactions with all the agents in the society [6, 19].

Random network: Erdös and Renyi have studied the properties of random graphs and have demonstrated a mechanism for generating random networks [29]. An undirected graph G(n,p) has n vertices in which the edges are connected to each other with a probability p. The graph shown in Figure 1 (b) is a

random graph with 20 vertices and the probability that an edge is present between two vertices is 0.2. It should be noted that the random network becomes fully connected network when p=1.

Scale-free network: Nodes in a scale-free network are not connected to each other randomly. Scale-free networks have a few well connected nodes called hubs and a large number of nodes connected only to a few nodes. This kind of network is called scale-free because the ratio of well connected nodes to the number of nodes in the rest of the network remains constant as the network changes in size. Figure 2 is an example of an Albert-Barabasi scale-free network where the size of the network is 50.

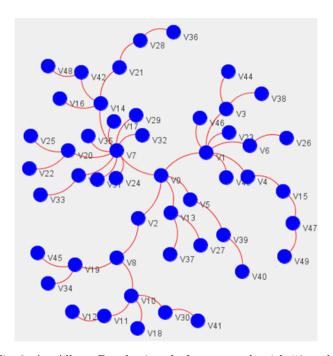


Fig. 2: An Albert-Barabasi scale-free network with 50 nodes

Albert and Barabasi [30] have demonstrated a mechanism for generating a scale-free topology based on their observations of large real-world networks such as the Internet, social networks and protein-protein interaction networks [31]. They have proposed a mechanism for generating scale-free networks based on the preferential attachment of nodes. At a given time step, the probability (p) of creating an edge between an existing vertex (v) and the newly added vertex is given by the following formula:

$$p = (degree(v)) / (|E| + |V|)$$

where (|E| and |V| respectively are the number of edges and vertices currently in the network (counting neither the new vertex nor the other edges that are being attached to it).

One may observe that the network shown in Figure 2 has a few well connected nodes, which are called hubs, e.g. vertices V7, and V1. A large number of nodes are connected to very few nodes. Scale-free networks exhibit a power law behaviour [30] where the probability of the existence of a node with k links (P(k)) is directly proportional to $k^{-\alpha}$ for some α .

Some characteristics of networks: Researchers have studied several characteristics of networks such as diameter (D), average path length (APL), degree distribution (k), clustering coefficient (C) etc. For our experiments we have used three of these characteristics whose definitions are given below.

- Degree distribution (k): The degree of a node in an undirected graph is the number of incoming and outgoing links connected to particular node.
- Average Path Length (APL): The average path length between two nodes is the average length of all possible paths between two nodes.
- Diameter (D): The diameter of a graph is the longest path between any two nodes.

3 Role Model Agent Mechanism

In this section we describe a mechanism that facilitates norm emergence in an agent society. We have experimented with agents that play the Ultimatum game. The context of interaction between the agents is the knowledge of the rules of the game. This game has been chosen because it is claimed to be sociologists' counter argument¹ to the economists' view on rationality [4]. In this context, when agents interact with each other, their individual norms might change. Their norms may tend to emerge in such a way that it might be beneficial to the societies involved.

3.1 The Ultimatum game

The Ultimatum game [33] is an experimental economics game in which two parties interact anonymously with each other. The game is played for a fixed sum of money (say x dollars). The first player proposes how to share the money with the second player. Say, the first player proposes y dollars to the second player. If the second player rejects this division, neither gets anything. If the second accepts, the first gets x-y dollars and the second gets y dollars.

¹ Sociologists consider that the norms are always used for the overall benefit of the society. Economists on the other hand state that the norms exist because they cater for the self-interest of every member of the society and each member is thought to be rational [32]. When Ultimatum game was played in different societies, researchers have observed that the norm of fairness evolved. As the players in this game choose fairness over self-interest, Sociologists' argue that, this game is the counter argument to economists' view on rationality.

3.2 Description of the multi-agent environment

An agent society is made up of a fixed number of agents. They are connected to each other using one of the social network topologies (fully connected, random or scale-free).

Norms in the agent society Each agent in a society has an internal norm. Each agent also has a norm to represent its maximum and minimum proposal and acceptance values when playing the ultimatum game. This norm is called as the personal norm (P norm). A sample P norm for an agent is given below where min and max are the minimum and maximum values when the game is played for a sum of 100 dollars.

- Proposal norm (min=1, max=30)
- Acceptance norm (min=1, max=100)

The representations given above indicate that the proposal norm of an agent ranges from 1 to 30 and the acceptance norm of the agent ranges from 1 to 100.

The proposal norm initialized using a uniform distribution within a range of 1 to 100, is internal to the agent. It is not known to any other agent. The agents in a society are initialized with an acceptance norm that indicates that any agent which proposes within the range specified by the norm will be accepted. The agents are only aware of their acceptance norms and are not aware of the acceptance norms of the other agents. In order to observe how proposal norms emerge, we assign a fixed value for acceptance norm to all the agents in the society. The acceptance norm of a society is given below.

- Acceptance norm (min=45, max = 55)

3.3 The norm emergence mechanism

The role models are agents who the societal members may wish to follow. The inspiration is derived from human society where one might want to use successful people as a guide. Any agent in the society can become a role model agent if some other agent asks for its advice. The role model agent represents a role model or an advisor who provides normative advise to those who ask for help. In our mechanism, each agent will have atmost one leader.

An agent will choose its role model depending upon the performance of its neighbours. We assume that agents that are connected know each other's performances. This is based on the assumption that people who are successful in the neighbourhood are easily recognizable. We argue that their success can be attributed to their norms.

Autonomy is an important concept associated with accepting or rejecting request to become a leader. When an agent is created, it has an autonomy value between 0 and 1. Depending upon the autonomy value, an agent can either accept or reject a request from another agent. Once rejected, an agent will contact the

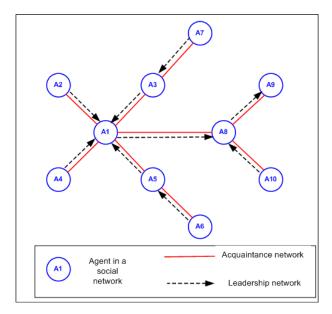


Fig. 3: Two layers of networks used in role model agent mechanism

next best performing agent amongst its neighbours. Autonomy of an agent is also related to accepting or rejecting the advice provided by the leader agent.

Assume that agent A and B are acquaintances (are connected to each other in a network). If agent A's successful proposal average is 60% and agent B's successful proposal average is 80%, then agent A will send a request to agent B asking for its advice. If agent B accepts this request, B becomes the role model of agent A and sends its P norm to agent A. The agent is autonomous to choose or ignore the advice depending upon its autonomy. When agent A decides to follow the advice provided it modifies its P norm based on the advice received from its role model agent.

Figure 3 depicts the two layers of networks that are used in our mechanism. The circles represent agents. The solid lines represent the social link network also known as an acquaintance network.

In our mechanism, an agent plays a fixed number of Ultimatum games with each of its neighbours (agents that are linked to it). In total, highly connected agents play more games than the poorly connected agents. Highly connected agents benefit from playing more games because they retain their competitive advantage of obtaining a wide range of information or norms from the agents that they are connected to while the poorly connected agents rely on the information from one or two agents that they are connected to. A highly connected agent is more likely to know about the best norm earlier than the poorly connected agent.

After one iteration, every agent looks for the best performing player in its neighbourhood. After finding the best performing player, the agent sends a re-

quest to the player requesting the agent to be its role model or leader. If the requested agent decides to become the role model, it sends its P norm (normative advice) to the requester (follower agent). The follower agent modifies its norm by moving closer to the role model agent's norm. The dotted line with an arrow (directed line) represents the leadership network that emerges at the end of interactions. In Figure 3, A1 is the leader of A2,A3,A4 and A5. Arrows from these four agents point to A1. This new kind of network that emerges on top of the acquaintance network is called a leadership network.

4 Experiments and results

In this section we present the experiments that we undertook to demonstrate that our mechanism leads to complete norm emergence when tested on top of different kinds of network topologies.

4.1 Norm emergence on top of random and scale-free networks

The role model agent based mechanism norm propagation was evaluated using Erdös-Renyi (ER) random network and Albert-Barabasi (AB) scale-free network.

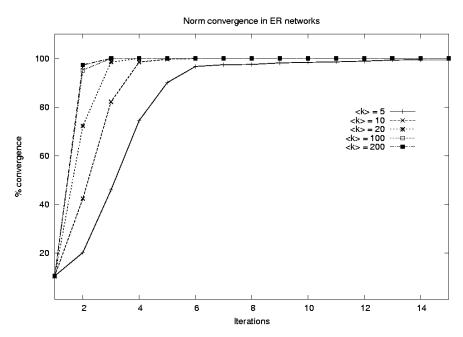


Fig. 4: Norm emergence in ER networks when average degree of connectivity is varied

At first we studied the effects of changing the average degree of connectivity (<k>) on norm emergence, while maintaining a constant population size (N). We varied the degree of connectivity for the ER and AB networks with N=200 . It can be observed from Figure 4 that as <k> increased the rate of convergence increased in both kinds of networks. When <k> is 10, 100% norm emergence was observed in the 6th iteration while it only took 3 iterations for convergence when the value of <k> is 200. Note that when <k> equals N, the network is fully connected, hence the convergence is faster. Similar results were also observed for AB networks (not shown here).

The comparison of ER and AB networks for the same values of N and <k> is shown in Figure 5. It can be observed that there is no significant difference in the rate of convergence in ER and AB networks. Our experimental results

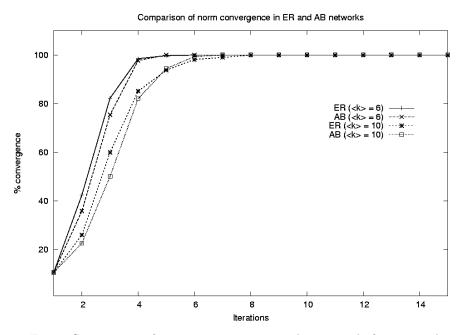


Fig. 5: Comparison of norm emergence in random vs scale-free networks

on norm convergence are in agreement with the statistical analysis carried out by Albert and Barabasi on the two kinds of networks [34]. They have observed that the diameter (D) and average path lengths (APL) of both the networks are similar for fixed values of N and <k>. The diameters of ER and AB networks, when N and <k> are fixed are directly proportional to log(N). As the diameters of both the networks are the same, the rate of norm convergence are similar.

The parameters D and APL of these networks decrease when the average connectivity of the network increases. When the average connectivity increases,

it is easier for an agent to find a leader agent whose performance scores are high. If the average connectivity is low, it would take an agent a few iterations before its leader obtains the norm from a better performing agent. This explains why norm convergence is slower when average connectivity <k> decreases (shown in Figure 4).

Even though the norm emergence properties of both kinds of networks are comparable, it can be argued that the scale-free network is better suited to model norm propagation because in the real world, people are related to each other through the social groups that they are in, such as the work group and church group. Information percolates among the members of the group through interactions. Also, people seek advice from a close group of friends and hence information gets transmitted across social network. Other researchers have demonstrated that scale-free networks are well suited to explain mechanisms of disease propagation [26] and dissemination of ideas [25]. Scale-free networks are more robust than random networks when random nodes start to fail and this phenomenon has been observed in real world networks [35].

Recently [36], it has also been observed that the diameter and average path lengths of an AB network depends upon the value of m. m is a constant that indicates the number of nodes to which a new node entering the network should be connected to, using the preferential attachment scheme. When m=1, D and APL are directly proportional to $\log(N)$ and for m>1, D is directly proportional to $\log(N)/\log(\log(N))$. In this light, Albert and Barabasi have suggested that the scale-free networks should be more efficient in bringing nodes closer to each other which will be suitable for propagation of ideas and norms.

4.2 Power law behaviour of the leadership network

We have also observed that the leadership network that emerges on top of the AB network follows power law behaviour. It is interesting to note that the leadership network that emerges on top of ER network follows power law behaviour when the average degree of connectivity is small. For smaller probabilities (p=.05, .1) we have observed that there are fewer leader agents with large number of followers and a large number of leaders with a few followers. Figure 6 shows the log-log plot of leaders with k followers in the x-axis and the number of leaders with k followers N(k) divided by the number of leaders with exactly one follower (N1) in the y-axis. The trendline shows the approximate power law behaviour of the leadership network. The slope of the power law curve was -1.6. Our results are in agreement with that of Anghel et al. [37] who studied the emergence of scalefree leadership structures using minority game. In their work, an agent sends its game strategy to all the agents in its neighbourhood. There is no explicit notion of leadership as each agent maintains an internal model of who its leader is. In our work, each agent chooses its leader explicitly and the leader sends the norms only to its followers. Also, the agents in our model have the notion of autonomy which is more representative of a realistic society.

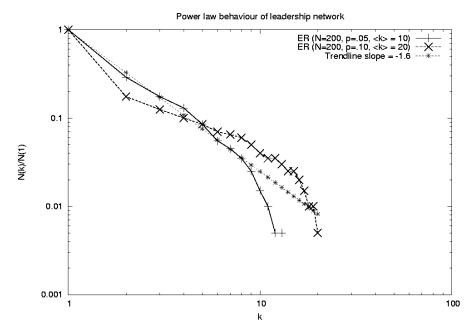


Fig. 6: Power law behaviour of the leadership network

5 Discussion

Our work is different (see Section 2.3) from other researchers in this area as we use the concepts of oblique transmission in the mechanism we have proposed. Verhagen's thesis [19] focuses on the spreading and internalizing of norms. This assumes that a norm is agreed or chosen by a top level entity (say, a Normative Advisor) and this group norm (G norm) does not change. The G norm is spread to the agents through the normative advice using a top-down approach. Our work differs from this work as we employ a bottom-up approach. In our approach the P norm evolves continuously. In his work, the P norm changes to accommodate the predetermined group norm.

The experiments described in this paper are our initial efforts in the area of norm emergence. The experiments are limited to a single agent society. We are interested in experimenting with scenarios that involve two or more inter-linked societies. In the real world, we attach more weight to a particular person's advice than others. Similarly, the weights of the edges (links) should be considered when the agent makes a decision on who to choose as a role model agent. We plan to incorporate this idea in our future experiments. Also, addition or deletion of links to a given topology have not been considered in the current mechanism. This is analogous to people relocating and forming new links. We have planned to experiment with our mechanism on top of dynamically changing networks. We also intend to demonstrate that our mechanism is scalable.

6 Conclusions

We have explained our mechanism for norm emergence in artificial agent societies that is based on the concept of role models. We have demonstrated the use of oblique norm transmission for norm emergence. Our mechanism was tested on top of three network topologies. We have shown through our experimental results that complete norm emergence can be achieved using our proposed mechanism. We have compared our work with the researchers in this area and also discussed the future work.

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