Vector Opinion Dynamics: An extended model for consensus in social networks

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Abstract

Most people hold a variety of opinions on different topics ranging from sports, entertainment, spiritual beliefs to moral principles. These can be based on a personal reflection and evaluation or on their interactions with others. How do we influence others in our social network and how do they influence us and how do we reach consensus? In this paper, we present our investigations based on the use of multiple opinions (a vector of opinions) that should be considered to determine consensus in a society. We have extended Deffuant model and have tested our model on top of two well-known network topologies the Barabasi-Albert network and the Erdos-Renyi network. We have implemented a two phase filtering process determining the consensus.

1. Introduction

An opinion represents a belief of a person based on her thoughts and ideas with regard to a specific subject. Most people will hold different opinions on different topics ranging from sports, entertainment, spiritual beliefs to moral principles. These can be based on a personal reflection and evaluation. However, many would actually form opinions based on their interactions with others especially when the information is hard to obtain [3]. These information sources include the media, advertising, family and friends, close associates (e.g. doctor, financial advisor) or even strangers.

The real-world that we are a part of forms has a particular network topology underneath. We are a part of different networks such as the network of our family (e.g. family tree), friends circle, work group and church group. These networks play an important role in the formation of opinions as interaction between the groups happen within these networks. Each member in the network directly or indirectly influences others in the network.

The researchers working on consensus formation are interested in finding why people form the opinions that they hold and how they influence each other. They are interested to know how many similar opinions does it take to reach consensus. They are interested in modeling how we compare opinions with one another and how it influences our decision making ability. For example, if two individuals are supporters of the same football team but support opposite political views; will they still interact with each other? These are a few of the classical questions explored in the field of opinion dynamics [2].

In this paper, we extend Deffuant's model of opinion formation by considering multiple opinions (a vector of opinions) and test it on top of two network topologies namely Barabasi-Albert's scale-free network topology and Erdos-Renyi's network topology.

The paper is organized as follows. Section 2 describes the background work on opinion formation and the associated models. The background information on network topologies are also presented in this section. Section 3 explains the proposed model and Section 4 presents the experimental results. The discussions are summarized in Section 5.

2. Background

In Section 2.1 we describe the concepts associated with opinion formation and also explain some of the well-known models of opinion dynamics. Section 2.2 provides some background information on network topologies that we have considered.

2.1 Concepts and models of opinion dynamics

Concepts such as social influence and opinion formation are the very essence of interpersonal behaviour, and they can entail a great deal of discussion as they are closely related to many other topics such as power, coercion, communication and group dynamics [3]. In spite of the plenitude of background information available, for the purpose of this study, we will only focus on related key concepts in the field of opinion dynamics, these are influence, consensus and polarization.

Opinion dynamics is a general term used to describe how and why opinions are formed and understand the conditions under which consensus or polarization is reached. *Influence* is a term that closely relates to the aforementioned, it can be defined as the ability to persuade others to share in a desired objective [2]. This involves modifying their attitudes, opinions, feelings and actions [3]. Influence in this paper will only refer to the modification of attitudes and opinions, since the modification of action is more of a psychological subject that relates to behaviour modification [3].

Another related term that is often used in conjunction with influence is *consensus*. Some consider it a special case of influence. Lecy and Sonne [4] describe it as the collective agreement of members of a group or community. Examples of consensus scenarios are many and diverse. They also point out that consensus can be explicit like it happens in elections. Voters have to explicitly show their agreement with a single opinion among many and therefore choose one candidate. In many other scenarios, it can be tacit especially if we are looking at 'cultures'. Individuals that belong to a certain socio-cultural group will often share many aspects of their lives. These are agreed on protocols that are not questioned by the people belonging to this culture. The way they greet each other, dress, types of food they like are a few examples of tacit consensus [3] .Consensus can be also happen within groups (phenomenon known as group consensus) regardless of the culture they might follow. A good example can be crowds that form into mobs, at times group members do not actually know the purpose or why they had joined the mob [4]. Thus, consensus is a broad term used to describe many scenarios of agreement.

Another phenomenon in opinion dynamics is the opposite of consensus, polarization. In communication and sociology, polarization refers to the division of social group into opposing subgroups that tend to fall within the extreme side of the spectrum, with less and less agents remaining neutral [5]. This makes it hard to reach consensus within two polarized groups.

The majority of studies on influence test their theories using computer simulations. Researchers in sociology, physics and computer science are collaborating to explain these dynamics better. Their simulations will have a number of agents (or participants) interacting with each other. Each agent will typically have one opinion that will be compared to another opinion based on the rules the researchers have established for the interactions. The reason why each agent (or participant) has a single opinion is to simplify the dynamics of interactions.

But, the degree of reality and modularity in these computer simulated models are debatable. These simple simulations can overlook some important aspects that could be investigated. This is especially true when the research community lacks a unified rule of thumb that determines the proportion of agreement needed to reach consensus. Does a pair of agents reach consensus when they are only 60 percent similar for example? Or do they have to have identical opinions to interact? In almost all the models analysed, the term consensus is used to imply that the two individuals or nodes are in a 100 percent agreement stage, which is hard to achieve realistically. This is recognised as a poor assumption by many of the studies; however, it is still prevalent [4].

Another weakness relating to modularity and reality in computer simulations is related to basing the experiments on a single opinion. In reality, individuals will interact even if they disagree on one opinion. They take into account more than a single opinion when making their decisions [2]. They compromise, negotiate and ignore some aspects of each other's opinions [3]. Thus, some of the recent models of opinion dynamics, such as Deffaunt's (2002), use agents that hold a vector (group) of opinions in their interactions to generate more practical results. However, these models are recent, which requires more research and extensions to the subject.

2.2 Network topologies

Networks are depicted as graphs thus making graph theory a significant field to address. Graph theory is the study of graphs in mathematics and computer science [6]. It is used to model relations between two or more objects. A graph in this context would typically refer to a collection of vertices or 'nodes' and a collection of edges that connect pairs of vertices, these edges can be directed or undirected. Two key graph models will be discussed as they are related to the research of this article. The models are the Erdős-Rényi (ER), and Barabási-Albert (BA) models.

Random graphs are graphs generated using a random process. The Erdős-Rényi (ER) model is the most prominent model for random networks. The model's algorithm takes the number of nodes as a parameter [7]. In order to construct the edges, users can either supply the desired number of edges or specify a probability of having an edge included (e.g. 60% chance). This model is an effective method if the researcher is designing a graph with minimal functionality yet has accurate graphical characteristics, or if they are creating a basic graph that can be altered later on [7]. Figure 1 depicts a random network as envisioned by Erdős and Rényi where the probability of a link to be present between two nodes is 20%.

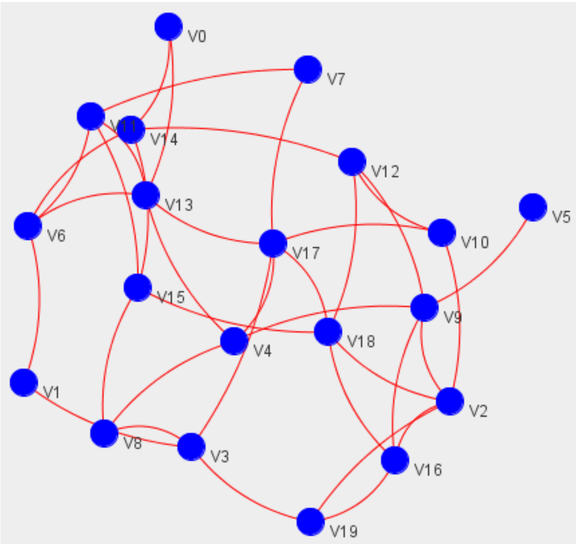

Fig. 1. ER network with link probability of 20%.

The second network topology is Barabási-Albert (BA) model. BA model [8] generates scale-free networks. It started as an extension to the ER model. It introduced some novel features to networks such network growth and preferential attachment of vertices (Barabási and Albert 1999). It acknowledges that vertex connectivity follows a scale-free power-law distribution. This network structure acknowledges that networks can expand and grow continuously. This is done by adding new vertices to the network. The new vertices are more likely to be attached preferentially to well-connected nodes.

The advantages of this model include, the more realistic presentation of real networks and the self-organising property of the network which allows it to organise itself efficiently especially in the event of growth.

In a scale-free social network, most individuals will be influenced by a small number of close associates, however, there will be a small number of individuals (known as seeds or hubs) that will influence a large number of other individuals. This is analogous to Pareto's 80/20 rule as Barabási points out (Barabási 2002). So, 80 percent of the links in the network will point to approximately 20 percent of the nodes. Refer to figure 3 for a visual representation of the network.

This paper will focus on examining a vector opinions based consensus model on top of well-grounded network topologies. The network topologies used are the Barabási-Albert and Erdős-Rényi models respectively. This is achieved through developing an executable tool that will simulate the following two questions, (1) how will agents interact when they have a vector of opinions attached to each one of them? Is it different to having a single opinion? And (2) Will the added network structure (i.e. Barabási-Albert and Erdős-Rényi networks) affect these interactions and how?

3. Extended model for opinion dynamics

Our model is the extended version of Defuuant's model. In the model proposed by Deffaunt et al. [1] agents interact randomly with other agents whenever the difference in the opinion values of two interacting agents is below a given threshold (*t*). It can be inferred from their results that high thresholds yield convergence towards an average opinion, whereas low thresholds result in several smaller opinion clusters.

3.1 Deffuant's model

The Deffaunt model is simple and intuitive. The model is populated with a set of N agents where each individual has a single opinion. The opinion is randomly chosen real number, and falls between 0 and 1. For example, agent x can have an opinion of 0.5 and agent y can have an opinion of 1 on the same subject. Agent y has a stronger opinion on the subject, while if agent x had an opinion of 0, they will have no opinion on the subject.

Agents will interact to observe the dynamics of opinion formation. Agents are picked at random, to re-adjust their opinions. If the difference in opinion is below the specified threshold (*t*), adjustment will occur. The threshold is essential since the study is based on the rationale which states that agents (and individuals, for that matter) will not even bother to interact unless their opinions are already close enough to start with [1]. The authors specify the adjustment formula as follows, please note that μ is the convergence rate parameter which ranges from 0 and 0.5 and x' and y' are the new opinion values of two interacting agents.

$$x' = x + \mu(y - x)$$
$$y' = y + \mu(x - y)$$

The results of the simulations demonstrated higher rates of convergence when the differences in opinions are closer to *t*. All interacting agents reached consensus at *t*=0.5, while several clusters of smaller converging opinion groups were formed when d equals 0.3. The existence of extremists who were not willing to change was also inevitable; they appeared when the threshold was close to 0 or 1.

The Deffaunt model also covers the notion of vector opinions. The conducted experiment dealt with agents having a vector of opinions. Each agent had 13 opinions. These opinions have a random real number value that falls between 0 and 1(like the individual ones). These opinions are limited to a set of pre-specified topics. An agent can interact with another agent if the calculated distance between their vector opinions lie below the threshold (*t*). This distance is calculated using the hamming distance [1].

Let's assume that an agent has a vector of m opinions, the hamming distance between its opinions and some other agent's opinion will be the number of different bits among the two vectors. So, x has the following opinions [0.1, 0.3, 0.2], and agent y has these opinions [0.1, 0.3, 0.3]. The hamming distance between them will be 1 since the opinion in the third position is the only difference. If *t* (threshold) was to be set to 1, then the agents with the opinions mentioned before will interact and adjust their opinions since the hamming distance between their opinions is 1. The adjustment process in this case is similar to the one conducted for the 'one opinion' experiment.

The results of this experiment conform to the results of the single opinion experiment. When the threshold is greater than 7, agents will reach consensus. This means that each of these agents agree on more than 7 topics of the 13 they posses. This is similar to the threshold from the single opinion experiment. Agents reached consensus when the difference in opinions was less than 0.5. When d is between 7 and 4, convergence occurs, however there are a few clusters or isolated opinions that are not connected to the main stream of converged opinions. Again, this is significantly similar to the single opinion experiment when *t*=0.3 [1].

One problem with the use of hamming distance metric is that it checks for an exact match. This can be useful if the opinion is regarded as binary, "yes" or "no" type of opinion. But, if the opinion can have many answers such as "hate", "love" or "sometimes", the difference cannot be seen as an exact match.

Another issue is that in real life, two interacting agents do not usually interact randomly as experimented by Deffuant et al. Agents belong to certain social structures such as family hierarchies and work place hierarchies. They interact with those agents with whom they are connected with based on the network topology. Agents in these structures influence each other's opinion.

3.2 Our extended model

We have made four extensions to the Deffuant's model. The extended model focuses on (1) using a vector of opinions where each opinion can be represented using a real number value and applying the Euclidean distance metric to compute the differences in opinions instead of Hamming distance (2) applying the Deffaunt model to well grounded network structures such as BA and ER network topologies, and (3) implementing a two phase filtering process to distinguish the similar agents based on the previous opinions.

The model has 100 agents with a vector of six opinions attached to them interacting in either a BA or an ER network. An adjustment of opinions will occur as a result of these interactions. At each time step two nodes will interact if a link connects the two agents based on the network topology. Each agent can access to an array of 6 elements representing 6 opinions. The opinions of the first 5 elements in the array are compared using the Euclidean distance which we will refer to as the difference in previous opinions. Let us take the opinion values for agent 1 to be x_i and the other to be y_i. The formula that computes the difference in opinions between two agents is given below.

$$\sqrt{\sum_{i=0}^{5}(x_i - y_i)^2} \,/\, \sqrt{5} \qquad \textbf{... 1}$$

Formula 1 shown above has a denominator of square root of 5, in order to obtain a normalized value for the difference in opinions in agents which lies between 0 and 1.

The adjustment of opinions will occur as a result of these interactions using the formula given by Deffuant et al (2002).

$$x_6' = x_6 + \mu(y_6 - x_6)$$
$$y_6' = y_6 + \mu(x_6 - y_6) \qquad \text{... 2}$$

μ in this case is the convergence parameter which can be changed for the simulations but will fall between 0 and 1. The value of t can be varied between 0 and 1. x_6' and y_6' represent the adjusted values. This process is iterative and it is repeated for 100 iterations.

We call this one stage filtering as single phase filtering process. The single phase filtering mechanism models the process of seeking advice from friends. Usually, our best friends have values that are closer to the ones that we hold. If we have to ask a suggestion on a new topic which is the sixth opinion in the experiment described, (e.g. which Tennis coach to choose, which school to choose for the kids), we might want to give a higher weightage to the past experience that we have had with the person whom we are choosing to seek advice. In this case, we usually ask our good friends. This corresponds to choosing people who have smaller differences in 5 opinion values.

The two phase opinion filtering process starts by checking if the difference in opinions is below the given interaction threshold (*t1*[1]) for the first 5 opinions. If the difference is below the threshold, we move to the next level of filtering. We then check if the difference in the sixth opinion for both agents is also below the personal threshold (*t2*). If it is the case then the nodes will adjust their sixth opinions according to Deffaunt's formula shown in formula 2. Otherwise, the nodes will not interact.

[1] Note that the variable *t* in single filtering mechanism corresponds to *t1* in the two phase filtering mechanism.

4. Experiments and results

4.1 Experiment 1 – The role of network topology on the convergence of opinions (single phase filtering process).

We have experimented with the role of network topologies on opinion convergence. In this experiment, μ and t were set to 0.5. There were 100 agents in the population. Each agent will change its opinion on the sixth opinion if the difference in the Euclidean distance between those two agents for the first five opinions are below *t*.

We have varied the diameter of the BA network, keeping the initial set up of the opinions for each agents, the same. The diameter of a graph is the longest path between any two nodes. A graph with large diameter (*d*) is a sparse graph where the average number of connections between agents is low. A graph with large number of connections will have a small value of *d*. We have experimented with three different values of *d* (*d*=4, 7, 13). Figure 2 shows the convergence of opinions for different values of d. The scatter plot shows the initial 6th opinion of all the agents (solid diamonds). Then, the agents that were connected to each other, modified their 6th opinion values. The experiment was run for 100 iterations. At the end of the experiment, the final 6th opinion values were plotted on the scatter plot. It can be observed that when the diameter of the network becomes small (*d*=4), then the opinions converge to a particular value (asterisks). When d was 13, there were large number of agents that did not converge to a single opinion (solid rectangles), however there are agents which have converged to a single value (0.56). When the diameter was set to 7 (solid circles), the opinion convergence improved. Though most of the agents had converged there are still few outliers that have not converged.

Similar results were observed for ER networks (not shown here). When the diameter of the network decreases, the average number of connections between the nodes increases (i.e the degree of connectivity increases). This speeds up the process of opinion convergence.

When the threshold (*t*) for adjusting the opinions decreases, the convergence decreases. By varying *t* in the experiment, different society models can be experimented. Low values for *t* correspond to seeking opinions from people (i.e. best friends) whose values match closer to the agent who is seeking advice and when the value of *t* is high, an agent chooses a range of friends including the best friends.

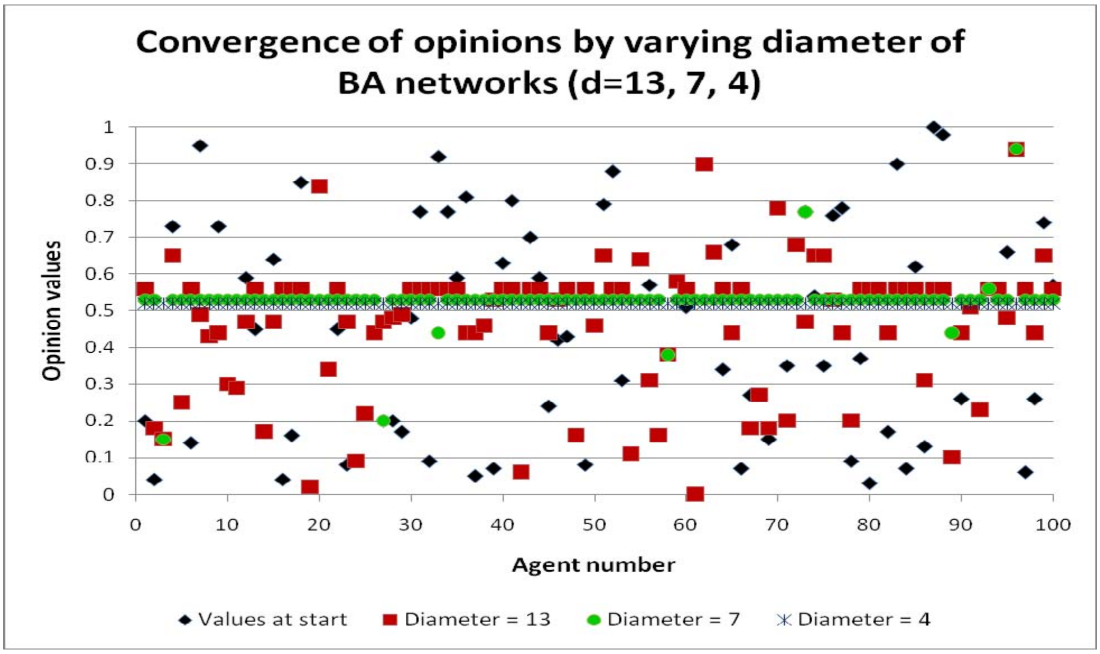

Fig. 2. Scatter plot of the convergence of opinions by varying diameter of BA network.

4.2 Experiment 2 – Comparison of opinion convergence in BA vs. ER networks (single phase filtering process)

The comparison of ER and BA networks for the same population size and initial values for the opinions connected using single filtering process on top of BA and ER network topologies is shown in Figure 4. It can be observed that there is no significant difference in the rate of convergence in ER and BA networks. Our experimental results on opinion convergence are in agreement with the statistical analysis carried out by Barabasi and Albert on the two kinds of networks [10]. They have observed that the diameter (*d*) of both the networks is similar for fixed values of population size and the degree of connectivity. The diameters of BA and ER networks, when population size and average degree of connectivity are fixed, are directly proportional to log (N). As the diameters of both the networks are the same, the rate of opinion convergence are similar.

The diameters of these networks decrease when the average connectivity of the network increases. When the average connectivity increases (low value of diameter), it is easier for an agent to find an agent that lies within the threshold value (the chances of finding an agent within the threshold is high). If the average connectivity is low, it would take an agent a few iterations before it finds an agent within the threshold value. This explains why opinion convergence is slower when diameter *d* decreases (shown in Figure 2).

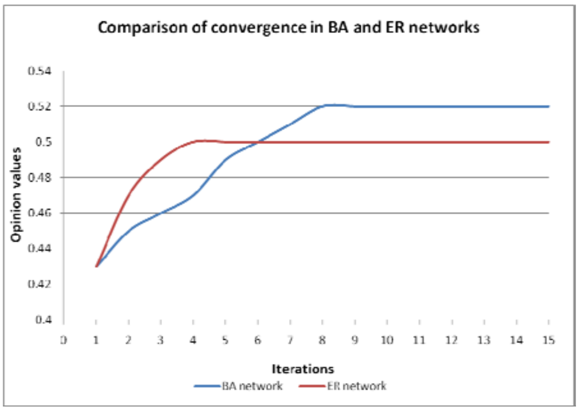

Fig. 3. Comparison of convergence in BA and ER networks.

Even though the opinion dynamics of both kinds of networks are comparable, it can be argued that the scale-free network is better suited for opinion propagation because in the real world, people are related to each other through the social groups that they are in, such as the work group and the church group. Information percolates among the members of the group through interactions. Also, people seek advice from a close group of friends and hence information gets transmitted across social network. Other researchers have demonstrated that scale-free networks are well suited to explain mechanisms of disease propagation and dissemination of ideas [9]. Scale-free networks are more robust than random networks when random nodes start to fail and this phenomenon has been observed in real world networks [11].

Recently [12], it has also been observed that the diameter and average path lengths of a BA network depends upon the value of m. m is a constant that indicates the number of nodes (N) to which a new node entering the network should be connected to, using the preferential attachment scheme. When m=1, *d* is directly proportional to log(N) and for m>1, *d* is directly proportional to log(N)/log(log(N)). In this light, Barabasi and Albert have suggested that the scale-free networks should be more efficient in bringing nodes closer to each other which will be suitable for propagation of opinions. Similar results in the context of norm propagation has been reported by Savarimuthu et al. [13].

4.3 Experiment 3 (Opinion convergence using two phase filtering process)

In the two phase filtering process, an agent adjusts its value for the sixth opinion, only if the Euclidean distance comparison of first 5 opinions between the two interacting agents is less than the threshold *t1* and the difference between the 6th opinion values is less than the threshold *t2*.

We believe that the two phase filtering mechanism mimics the real world. The second phase of filtering takes into account an agent's autonomy or the stubbornness on a particular decision (i.e. sixth opinion which is on a particular topic). The agent, who is deciding on a particular topic (sixth opinion), might want to give a higher weightage to its own opinion than others opinion (because the agent might be an expert in that field or simply because the agent feels that is the right choice).

For example, you might want to choose a Dentist based on the recommendation from your best friends. In that case you will attach a low threshold for *t1* (e.g. *t1*=0.2) while the value of *t2* can be high (e.g. 0.5). Alternatively, if you are in the medical field yourself, then you might want to choose the best Dentist based on the recommendation from all your friends (*t1*=0.5) and considering your own preferred dentist (high preference to the Dentist that you know might be good (*t2*=0.2)).

In essence, the two phase mechanism provides a way for associating priorities or weightages to recommendations from friends and one's own preferences. If *t1*=0.2 and *t2*=0.5 in a system, then this society will have high weightage for the recommendations from the best friends (*t1*=0.2) and the individual agent's opinion has a low weightage (*t2*=0.5). On the other hand if *t1*=0.5 and *t2*=0.2, then the agents in the society attach higher weightages to their own preferences.

Figure 4 shows the convergence of opinions in a BA network using the two-phase filtering process. It can be observed that convergence when t1=0.5 and t2=0.5 (solid triangles) is higher than when t1=0.3 and t2=0.5 (solid rectangles). This is because, the agents had opinions that were initialized using random distribution (0 to 1]. Such a society is loosely-knit, because of random initial values (solid diamonds) in the population. Loosely-knit society is a society with huge differences in values. For example, a society could be made up of well educated and poorly educated people. The well educated people might have certain political view while the uneducated people might have a different view. In a loosely-knit society agents usually interact within their own sub-groups and hence high weightages to opinions of like- minded people. In this case, there will not be a convergence in opinions to a single value. There will be a few clusters of opinions.

If the society was a well-knit society (e.g. people belonging to a religious group) where the difference between the 5 opinions of all the agents was less than *t1*, then the double filtering mechanism will result in the convergence to a single opinion.

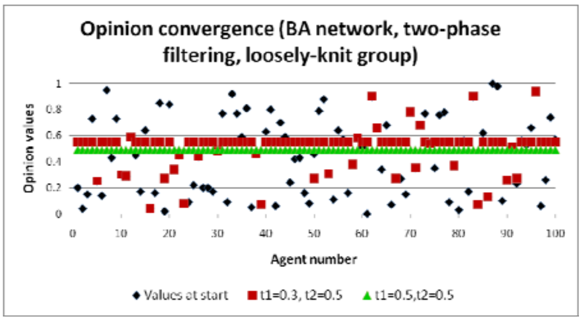

Fig. 4. Scatter plot of opinion convergence in a BA networking using two-phase filtering process in a loosely-knit group.

Figure 5 shows the inital set of values for the 6th opinion in a well-knit society (solid diamonds). The average values of the first 5 opinions is shown using solid triangles. Note that the triangles form a cluster (mean = 0.25) which represents a well-knit society. This case (*t1*=0.3 and *t2*=0.5) results in the 6th opinion convergence to a single value which is in contrast to the result obtained in experiment 3 for the same values of *t1* and *t2*.

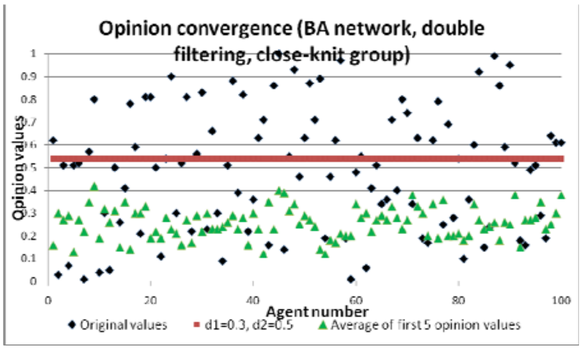

Fig. 5. Scatter plot of opinion convergence in a BA networking using two-phase filtering process in a well-knit group.

5 Discussion and future work

When two-phase filtering mechanism is compared with single-phase filtering mechanism, the rate of convergence to a single opinion is marginally faster in single-phase mechanism. It is quite intuitive that the two-phase mechanism produces marginally slower results because of the second filter that has been added on top of the single phase filtering mechanism. We believe that the two phase filtering mechanism provides an option for the experimenters to attach different weightages to other's opinion and individual's opinion while making decisions which is a useful improvement. The weightages can be changed to simulate different societies or population groups that have different preferences on thresholds.

The data obtained from the experiment 1 and 2 show evidence that network topology of a society is a key aspect in opinion dynamics. The diameter of a network influences the rate at which consensus is reached. We have shown that Deffuant's model can be tested on top of two network topologies (BA and ER networks). We have extended Deffuant's model by incorporating vector opinions which are compared using Euclidean distance measure instead of hamming distance. The results in this model acknowledge the fact that two agents can reach consensus even if their opinions are not exactly the same.

The two phase filtering mechanism proposed in this paper can be extended to match more realistic scenarios. Some of the proposed constructs for extension are 1) adding media sources to the model (e.g. how common knowledge obtained through television, radio, Internet and other multimedia sources can help opinion dynamics), b) implementing asymmetrical links and c) conducting investigations based on gathering real data from different regions or cultures (e.g. voting data in New Zealand) or branding data (e.g. opinion on iPhone in a loosely-knit vs. close-knit Internet societies).

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