Recomputing Historical Barometric Heighting

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Abstract

This paper discusses the method of determining heights of mountains during the original geodetic survey of Victoria. From 1840 to 1875, more particularly the 1860s, geodetic surveyors were charged with the responsibility of mapping the colony. The subject of this paper is their efforts to determine the elevations by barometric heighting. A brief introduction to other methods is given while particular attention is paid to the determination of the height of Mount Sabine in the Otway Ranges, Victoria, by Surveyor Irwin in 1865. Attempts are made to recompute his original observations.

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1 Introduction

During the 1860s the colony of Victoria was being mapped to manage land settlement and to determine land use. Surveyors were at the forefront; they were of course joined by people like Selwyn(geologist), Neumyer(scientist) and Daintree(engineer - photographer).

One of the primary tasks was to determine the heights of important mountains. It should be remembered from the outset that these were comparatively hard times. There were few roads, access was difficult if not arduous. Instrumentation was relatively simple; though it is true to say that the traditional surveying instruments were bolstered to meet the needs of the geological and magnetic surveys.

There were four primary methods for determining heights of relatively inaccessible mountains. Conventional spirit levelling and reciprocal vertical angles were known at the time but were infrequently used in these circumstances. This was due to the impracticalities and the lack of the accurate knowledge of atmospheric refraction.

First, height could be determined by the measurement of a baseline and angles (Figure 1).

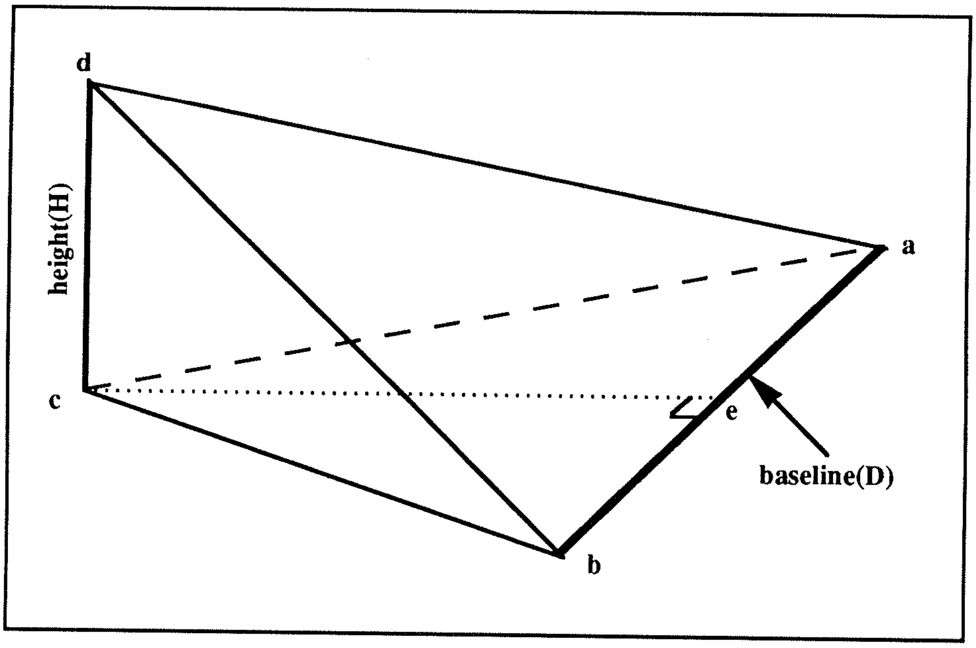


Figure 1

In Figure 1 the distance **ab**, horizontal angles **abc** and **bac** and vertical angles **cbd** and/or **cad** are measured. If both the vertical angles are observed, redundant measurements will provide two solutions for the height. These of course can be used as a check on any gross error or the two results may be meaned. The solution, however, may still be ambiguous. In Figure 1 the point **e** is shown as lying between **a** and **b**. It could lie on either extension of the line; this ambiguity can be resolved in the field by simple inspection(in most cases). Assuming the ambiguity has been resolved and only one vertical angle is observed (say at **a**), the height, **H** can be determined by;

$$H = \frac{D*Sin[cba]*Tan[cad]}{Sin[cab + abc]} \qquad (i)$$

The precision of the determination of H can be calculated by the partial derivative of the above equation. Assuming the following values;

D ≈ 2000m (standard deviation 0.2m),

horizontal angles ≈ 80° (standard deviation 20"),

vertical angle ≈ 6° (standard deviation 20").

the variance of a single observation of the height is approximately 0.18m. If 300 observations are taken the variance of the mean would be 0.01m! This precision may be optimistic for the 1860s but it provides an interesting comparison with the results for barometric heighting provided later in this paper.

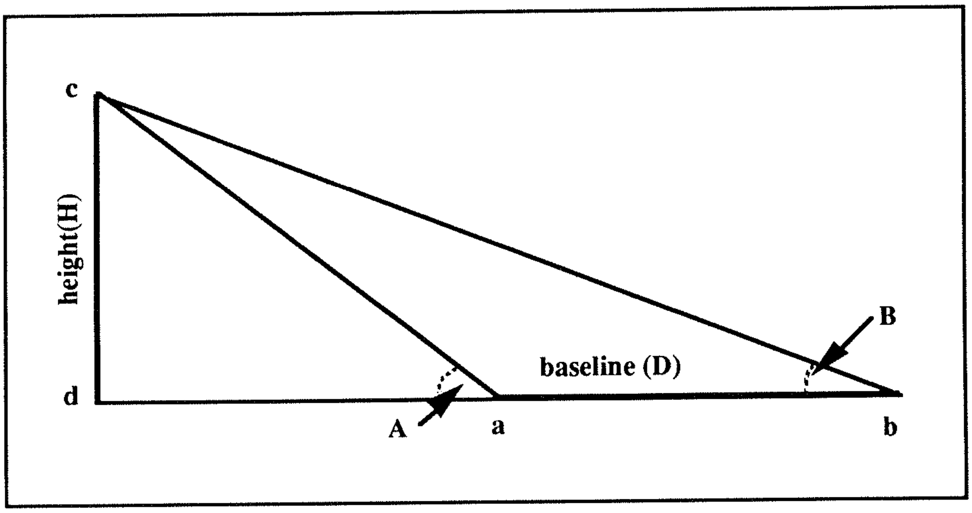


Figure 2

Second, the heights could be determined using the two angle baseline technique shown in Figure 2. Here the distance ab and angles A and B are measured and the height may be calculated from;

$$H = \frac{D*Tan[B]*Tan[A]}{Tan[B] - Tan[A]} \qquad (ii)$$

The above methods show a surprising precision for the height determination. It may therefore be difficult to understand why any other method would be selected. What the above mathematical analysis does not uncover is the systematic errors involved or the prevailing work conditions. Large systematic (or gross) errors, such as wrong identification of mountains or the tops of tall trees assumed to be the summit of the mountain, could quite easily produce results that are tens or even a 100 metres in error. These techniques were labour intensive and with the government more interested in land settlement and sale, topographic information, such as height, was given low priority.

Third, observed angle of depression to a sea horizon was a method used where such a horizon was visible. Although reliant on a knowledge of atmospheric refraction, the height could be determined from;

$$H = \frac{R}{2} * \frac{\mathrm{Cos}^2 z}{(1 - m)} \qquad \text{(iii)}$$

where
H = height above the visible sea horizon (ft),
R = mean radius of the earth at the point of observation (ft),
z = zenith angle (degrees),
m = coefficient of refraction.

There is no evidence of this formula being used; although in the Otway Ranges the surveyors, influenced by their maritime training, would have been aware of the method.

The first three methods are given as an introduction to the main technique employed by the surveyors. The methods are indicative of the efforts that were taken to avoid impractically slow spirit levelling and the difficulties of measuring distances. Angle measurement was always preferred to distance measurement; a trait that was to survive one hundred years until the advent of electromagnetic distance measurement (EDM) equipment during the late 1960s. More direct measurements of height, such as horizontal and vertical traverses and levelling, were possible. The performing of these tasks on heavily wooded mountain sides rendered them most impractical. It was far more expedient to measure baselines on flat open planes and then observe angles to mountain tops. This approach was both a practical and economical solution. There remained the ever bearing problem that heights, relative to a reference datum, were required. The first two methods derived heights relative only to the surrounding plane. Horizon depression angles (if ever used) were only applicable to coastal ranges. All these shortcomings obviously led the surveyors to adopt another method.

2 Barometric Heighting

A fourth method therefore, one without most of the above disadvantages, was barometric heighting. The method was simple to perform and could produce heights relative to a base station. By the 1860s this form of heighting was well understood as it had been used on many colonial surveys, not only in Australia but Canada [Thompson, p206, 1966], India and Africa. Baker [Baker, 1873, p157] describes barometric heighting in the following terms;

> *The method of finding the difference of levels ... by the barometer, though frequently recommended, will be found to fail in point of accuracy, on account of the sudden changes in the pressure ... . This method, therefore can never be relied upon further than as a rough approximation.*

His comments may be somewhat harsh but reflect a degree of uncertainty due to the differential change in pressure between two stations. So, even if it were simple and practical, there were (and continue to be) some known and respected disadvantages.

In 1865 J.C. Irwin was a surveyor under the superintendence of R.J.L. Ellery, Government Astronomer and in charge of the Geodetic Survey of Victoria. Besides prosecuting the survey in the Otways, Irwin was charged with determining the heights of the mountains. This latter task was important for the production of reliable topographic maps (as was the former of course) and was also linked (when close to the coastline) to the marine and coastal survey and to safe navigation.

Meteorological instruments available to Irwin would have included barometers [VPRS 780/7];

- Newman type
- travel instruments on Fortess's principle
- marine type
- boiling point apparatus, and
- aneroids.

The method was simple. Observe pressure and temperatures (instrumental and atmospheric) at two locations simultaneously and the relative height difference could be calculated from tables. If one of the stations was at sea level (or could be easily referred to it) then heights were then relative to a recognised datum. In Irwin's case, observations were taken from 1st July 1865 to 30th November 1865. In 362 cases he has provided the reduced height difference for these readings. His observations were taken at 9am, 3pm and 9pm. A base station was established on Cape Otway (Figure 3) to which the height difference was referred. His camp was approximately 60m (stated as 200feet) below the summit of Mt. Sabine. In general Irwin took the observations (Table 1) himself, though obviously only at one end of the line. There are noted occasions when the survey party's cook read the barometer. These observations should be regarded with some scepticism as Irwin noted [VPRS 780/7];

> *The readings from the 13th to the 24th should not have much dependence placed in them, as they were taken by my cook, and I notice some extraordinary jumps now and then.*

Irwin states that he used "Admiral Fitzroy's Mountain Barometer Tables" for reduction, and that the calculations were performed according to "Baily's formula". A search of mid 19th century texts has failed to uncover any such tables or the related formula[2]. These tables are no doubt similar to those provided in 1881 by the US Coast and Geodetic Survey [Johnson, p135, 1890].

What reliance can be placed upon Irwin's or similar results? As the formula is not available can the results be repeated?

3 Recomputation

A formula of interest, but not used in the recomputations is provided in Park [Park, p35, 1914]. It is of interest for two reasons; the simplicity of the formula (non-logarithmic), and Park is one of the very few early texts, written in New Zealand, on Surveying, coincidently written by a Professor of my present University. The formula is given as;

$$H = 49{,}000 * \left[\frac{(B-b)}{(B+b)}\right] * \left[1 + \frac{(T+t)}{900}\right] \qquad \text{(iv)}$$

where

H = difference in height between the two stations (feet),

B = reading in inches at the lower station,

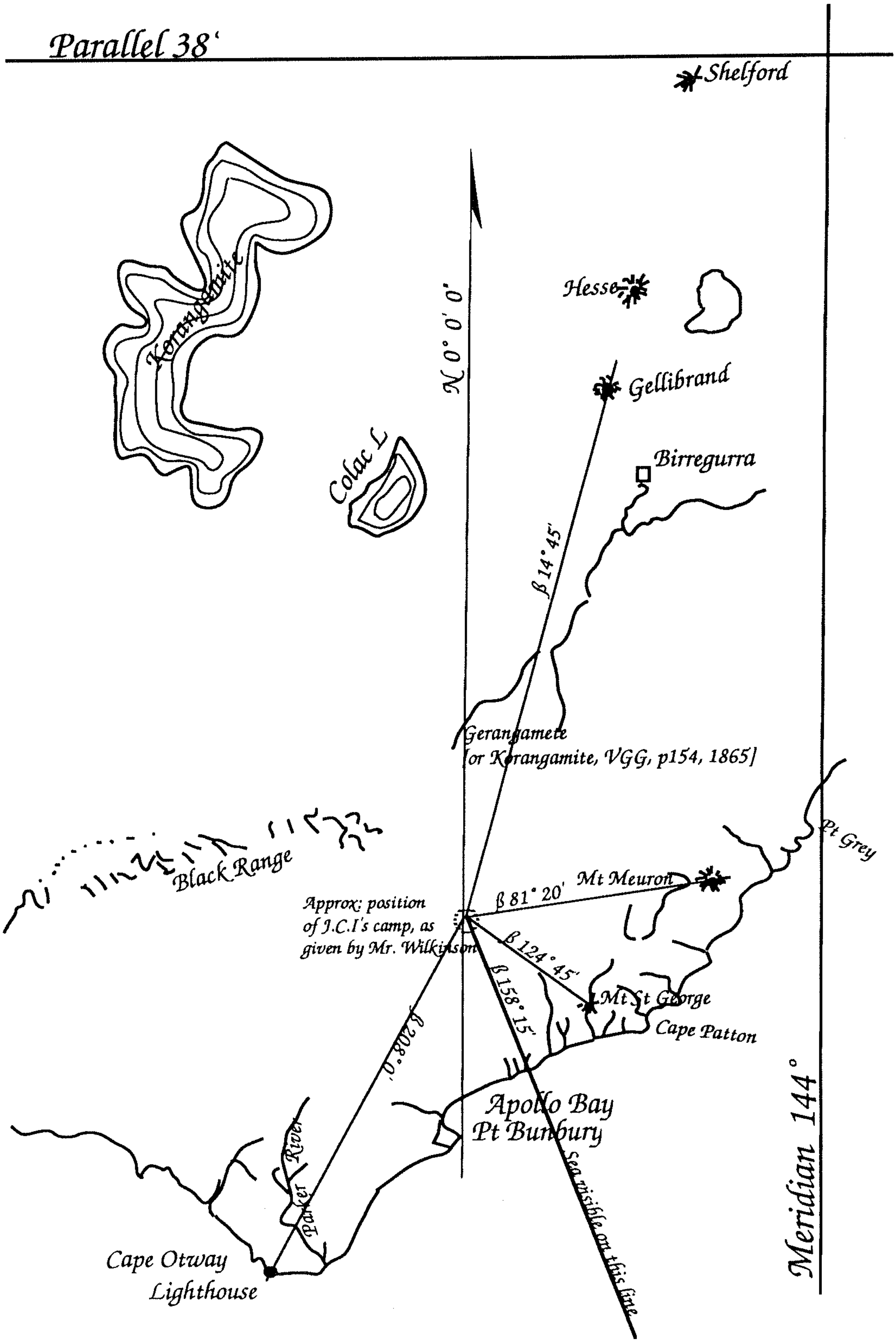
b = reading in inches at the upper station,

T = temperature at the lower station,

t = temperature at the upper station.

To understand and analyse the observations the data had to be re-reduced and an appropriate formula found. The following formulae were considered;

2 I would appreciate if anyone locates a reference to these texts that they may pass them on to me.



A facsimile version of Irwin's map of the district

Figure 3 (after Irwin, 1865 [VPRS 780/7]

1 Clendenning and Olliver [Kellie and Young, 1986, p246]

$H = H' + (H - H')_1 + C_1 + C_2 + C_3$ (v)

where

H = elevation of the station to be determined (in feet)

H' = elevation of the base station (in feet)

$(H - H')_1$ = preliminary elevation difference described by

$(H - H')_1 = K*(\log_{10}p' - \log_{10}p)$

where

p' = the observed pressure at the station to be determined (inches Hg),

p = the pressure at the base station (inches Hg), and

K = 18402.6*3.28084 (a constant with metric conversion)

$C_1 = (H - H')_1 * 0.00366*t$

where

t = mean atmospheric temperature (°C) of the two stations

$t = (t_1 + t_2)/2$

where

t_1 = air temperature at the base station (°C)

t_2 = air temperature at the station to be determined (°C)

(to be converted to °F for formula compatibility)

$C_2 = (H - H')_1*(0.00264*\text{Cos}2\theta + 0.00016*(H - H')_1*10^{-3})$

where

θ = latitude of observation (degrees).

$C_3 = (H - H')_1*0.375*(\frac{e}{p_v})$

where

e = partial vapour pressure at observation (mm Hg)

p_v = total vapour pressure (mm Hg)

$e = e' - 0.00066*p_T*(t_d - t')$

where

t_d = dry bulb reading (°C),

t' = wet bulb reading (°C),

e' = saturation bulb reading (mm Hg),

p_T = total air pressure (mm Hg).

2 Boyles and Charles [Bannister and Raymond, 1984, p82]

$$H = 18336.5*\log_{10}\left[\frac{p_2}{p_1}\right]*\left[1 + \frac{t_1 + t_2}{500}\right] \qquad \text{(vi)}$$

where

H = the difference in height (metres),

p_2 = the pressure at the upper station (mm Hg),

p_1 = the pressure at the lower station (mm Hg),

t_2 = the temperature at the upper station (°C),

t_1 = the temperature at the lower station (°C).

3 Single Base [O'Connor, 1957, p301]

$$H = K*T_m*\log_{10}\left[\frac{p_1}{p_2}\right] \qquad \text{(vii)}$$

where

H = the height difference between stations (metres),

K = a constant depending on the density of air at S.T.P., standard gravity, the density of mercury at 0°C, and units of measurement,

p_1 = is the pressure at the lower point (mm Hg),

p_2 = is the pressure at the upper point (mm Hg),

T_m = the mean air temperature between p_1 and p_2 (°C).

4 Guyot's Formula and Tables [Gillespie and Staley, 1902, pp276];

$$H = \log_{10}\left[\frac{p_1}{p'}\right]*60158.6 + C_1 + C_2 + C_3 \qquad \text{(viii)}$$

where

H = the height difference between the stations (feet),

p_1 = observed pressure at the lower station (inches Hg),

p' = observed pressure at the upper station reduced to the temperature of the barometer at the lower station, hence

$p' = p_2*[1 + 0.0008967*(T_1 - T_2)]$

where

p_2 = observed pressure at the upper station (inches Hg),

T_1 = the temperature of the barometer at the lower station (°F),

T_2 = temperature of the barometer at the upper station (°F).

$$C_1 = \left[\frac{t_1 + t_2 - 64}{900}\right]$$

where

t_1 = the air temperature at the lower station (°F),

t_2 = the air temperature at the upper station (°F).

$C_2 = 0.002608*Cos(2\theta)$

where

θ = the mean latitude between the stations (degrees).

$$C_3 = \left[\frac{H' + 52252}{20890790.50} + \frac{p_1}{10445395.25} \right]$$

where

H' = first estimate of height difference, hence

$H' = H - C_3$

and where the mean radius for the earth has been adopted from Ellery's 1891 paper [Ellery, 1891]. The mean radius as originally given by Gillespie and Staley [op. cit.] was 20886860 feet. Ellery's figure may not agree exactly with more modern values (say 20925722 semi minor axis [AMG Manual, 1972, p8]) but it does facilitate direct comparisons with Irwin's calculations.

The daunting task of re-computing 362 observation through four formulae was eloquently handled by utilising SPSS software on a Macintosh. SPSS was considered superior to a computational spreadsheet capable of separately computing all the above formulae, as it had the advantage of determining the associated statistical data. The original observations were input in the following format (Table 1) for re-computing.

day	**bar$_2$**	**t$_{21}$**	**t$_{22}$**	**bar$_1$**	**t$_{11}$**	**t$_{12}$**	**H**
79.375	30.069	53.0	53.8	28.401	46.0	45.5	1536.70
79.625	30.020	55.0	56.5	28.419	52.0	54.0	1503.10
79.875	29.965	53.0	52.0	28.399	47.0	45.0	1442.80
80.375	29.830	53.0	54.0	28.280	49.0	50.5	1451.20
80.625	29.731	57.0	60.0	28.201	58.5	62.0	1478.90
80.875	29.720	55.0	54.0	28.179	51.5	49.0	1447.00
81.375	29.815	53.0	54.0	28.161	48.0	49.5	1548.00
81.625	29.793	56.0	58.0	28.181	50.0	51.5	1515.40
81.875	29.770	53.0	52.5	28.181	42.5	41.0	1457.30
82.375	29.497	53.0	53.0	27.980	47.5	48.5	1427.10
82.625	29.432	56.0	55.0	27.921	58.0	62.0	1469.20
82.875	29.445	55.0	54.0	27.939	49.5	48.0	1419.70
83.375	29.604	54.0	51.0	27.979	48.5	48.0	1523.00
83.625	29.790	53.8	52.0	28.120	47.0	46.0	1551.50

....

Table 1

where

day = decimal days from the commencement of the observations,
bar_2 = barometer reading at Mt Sabine (inches Hg),
t_{21} = temperature of the barometer at Mt Sabine (°F),
t_{22} = air temperature at Mt Sabine (°F),
bar_1 = barometer reading at Cape Otway (inches Hg),
t_{11} = temperature of the barometer at Cape Otway (°F),
t_{12} = air temperature at Cape Otway (°F),
H = height difference as computed by Irwin (feet).

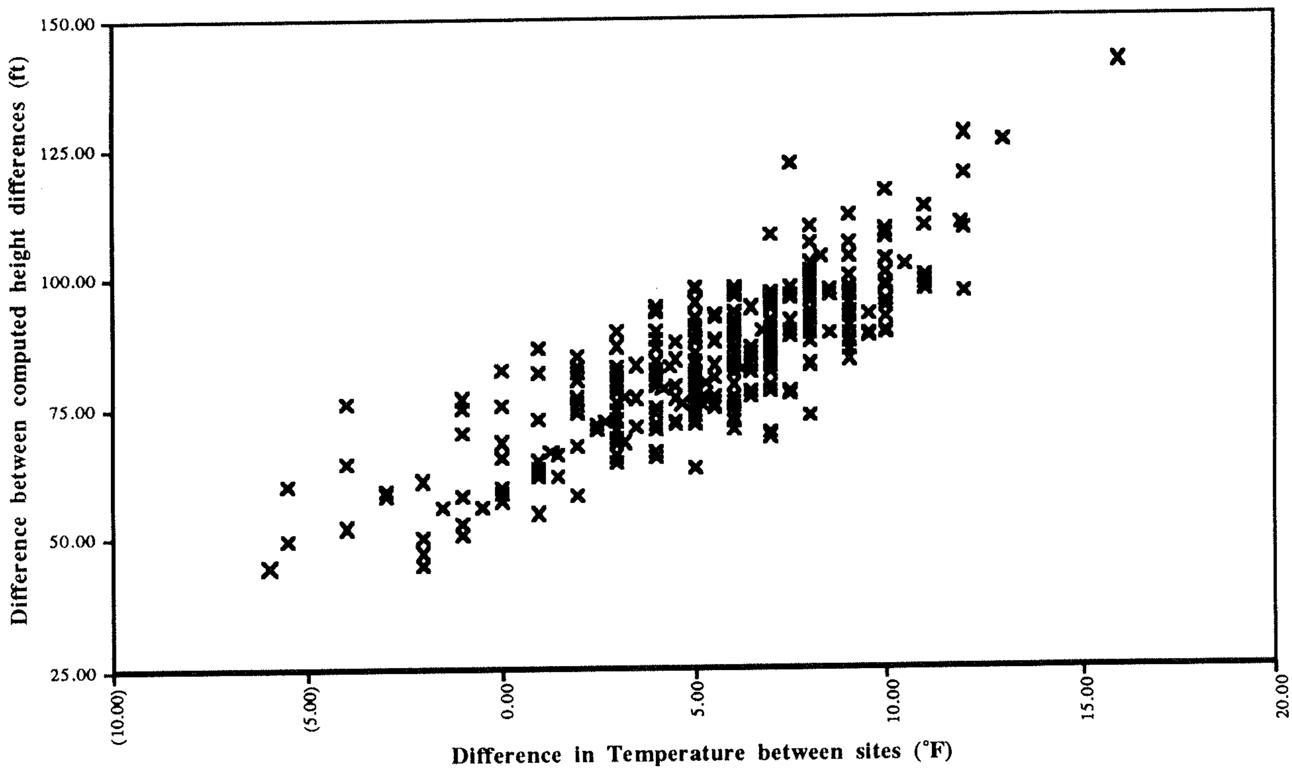


Figure 4

It was intended to discover the formula which best approximated that used by Irwin and to compute the mean and variance. Experiments were conducted using formulae 1, 2 and 3. The results were less than satisfactory. There were considerable variations when comparing the computed height differences. Figures 4 and 5 show typical results for part of this analysis. It would appear that there is little similarity between the first three formulae and that used by Irwin.

Figure 4 shows a scatter plot of the difference in site temperatures (Δt) against the difference in height differences (ΔH) between Irwin's calculation and that derived from formula 2; that is;

Δt = difference in temperature between sites (°F),

$$\Delta t = t_1 - t_2 \qquad \text{(ix)}$$

where

t_1 = air temperature at Cape Otway (°F),

t_2 = air temperature at Mt Sabine (°F).

$$\Delta H = H_I - H_{F2} \qquad \text{(x)}$$

where

H_I = height difference as computed by Irwin (feet),

H_{F2} = height difference as computed from formula 2 (feet).

There is a distinct relationship between the height difference and temperature difference. While it is true that there is a relationship between height and temperature there should be no relationship between the differences of differences. That is, ΔH should not vary in a correlated way with Δt. This apparent correlation is assumed to be due to some systematic error or different application of the temperature as a variable in the two equations. The correlation coefficient was 0.8601 at a significance level of less than or equal to 0.01.

Figure 5 shows a scatter plot of days from commencement of the survey against the difference in height differences (ΔH) between Irwin's calculation and that derived from formula 3; that is;

$$\Delta H = H_I - H_{F3} \qquad \text{(xi)}$$

where

H_I = height as computed by Irwin,

H_{F3} = height as computed from formula 3.

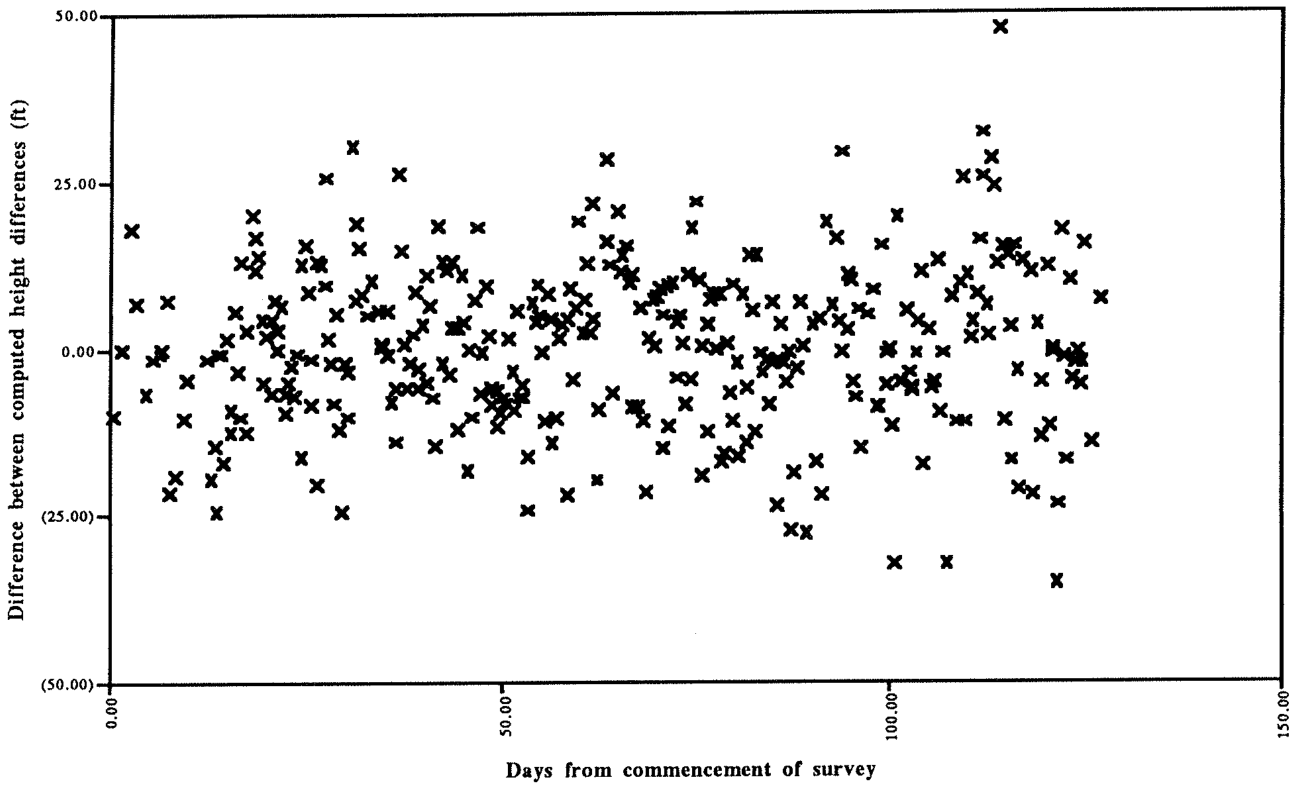


Figure 5

The large spread in the values is compelling evidence that there is little direct relationship between the two formulae.

Finally, Guyot's Formula was applied and as the following figures demonstrate, the results were most favourable. After an initial comparison the original 362 cases were subjected to a standard SPSS box test for the elimination of extreme and outlier cases. This was performed by calculating ΔH and reducing the set by successively discarding the exceptional values. Table 2 shows the statistical results of this procedure;

Iteration	mean	median	variance	skewness	# cases rejected
1	-5.25	-3.31	389.16	-11.25	17
2	-3.49	-3.30	1.86	-1.74	6
3	-3.35	-3.24	1.12	-0.71	3
result	-3.25	-3.24	1.12	-0.71	-

Table 2

This procedure left 326 cases. Figure 6 shows a scatter plot comparing Irwin's height differences with the height differences computed from Guyot's formula.

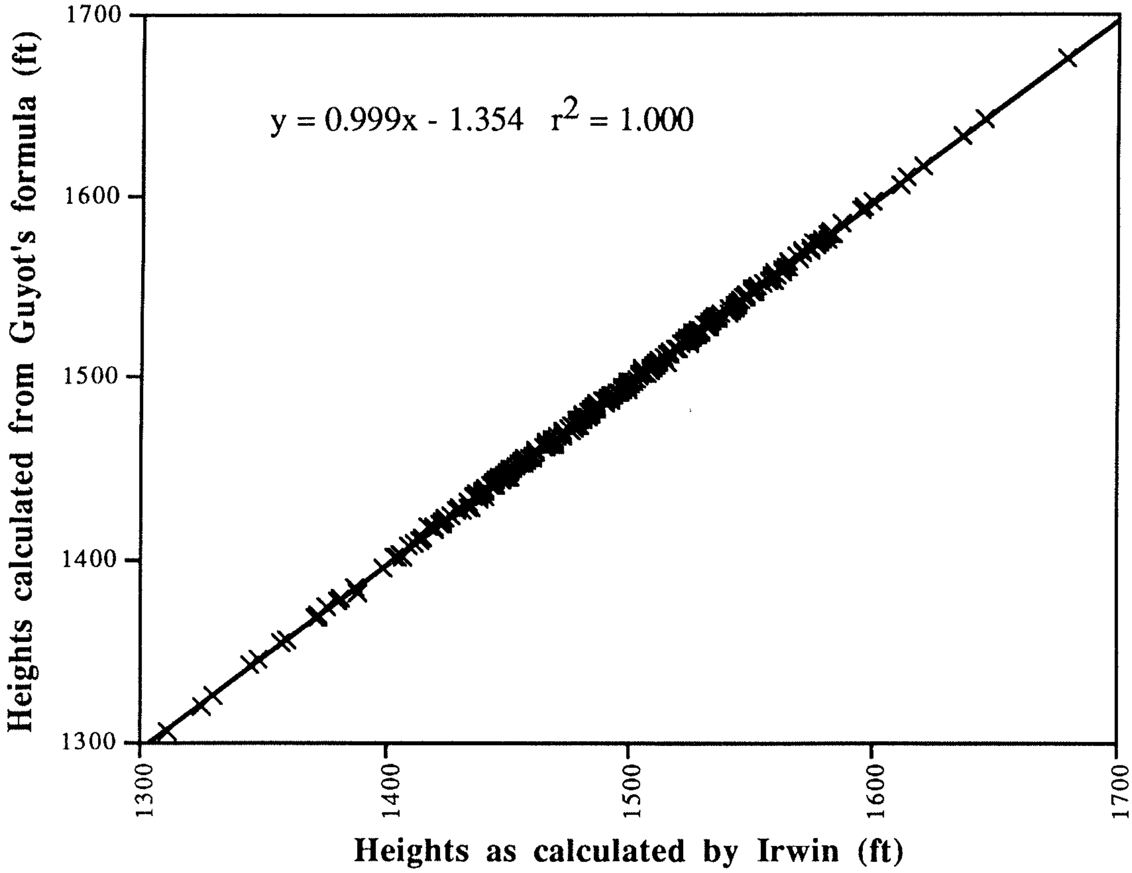


Figure 6

There is a high level of correlation between the two variables ($r^2 = 1.000$). The final values for the computed and Irwin's height differences (feet) are;

method	mean (ft)	variance (ft^2)
computed with Guyot's formula	1490.94	3116.316
computed by Irwin	1494.29	3123.531

A T-test on the two data sets resulted in;
T value of -57.09, and

difference (ft)	std. dev. (ft)	stand. error (ft)
3.35	1.059	0.059

(two tail prob = 0.000)

Figure 7 represents a histogram plot of the two final height difference variables. Given the above tests it would seem the two formulae are identical except for a shift in the means of 3.35 feet. This shift is possibly due to a linear shift and displayed no correlation with any of the other variables, that is, not with pressure, temperature, height nor time.

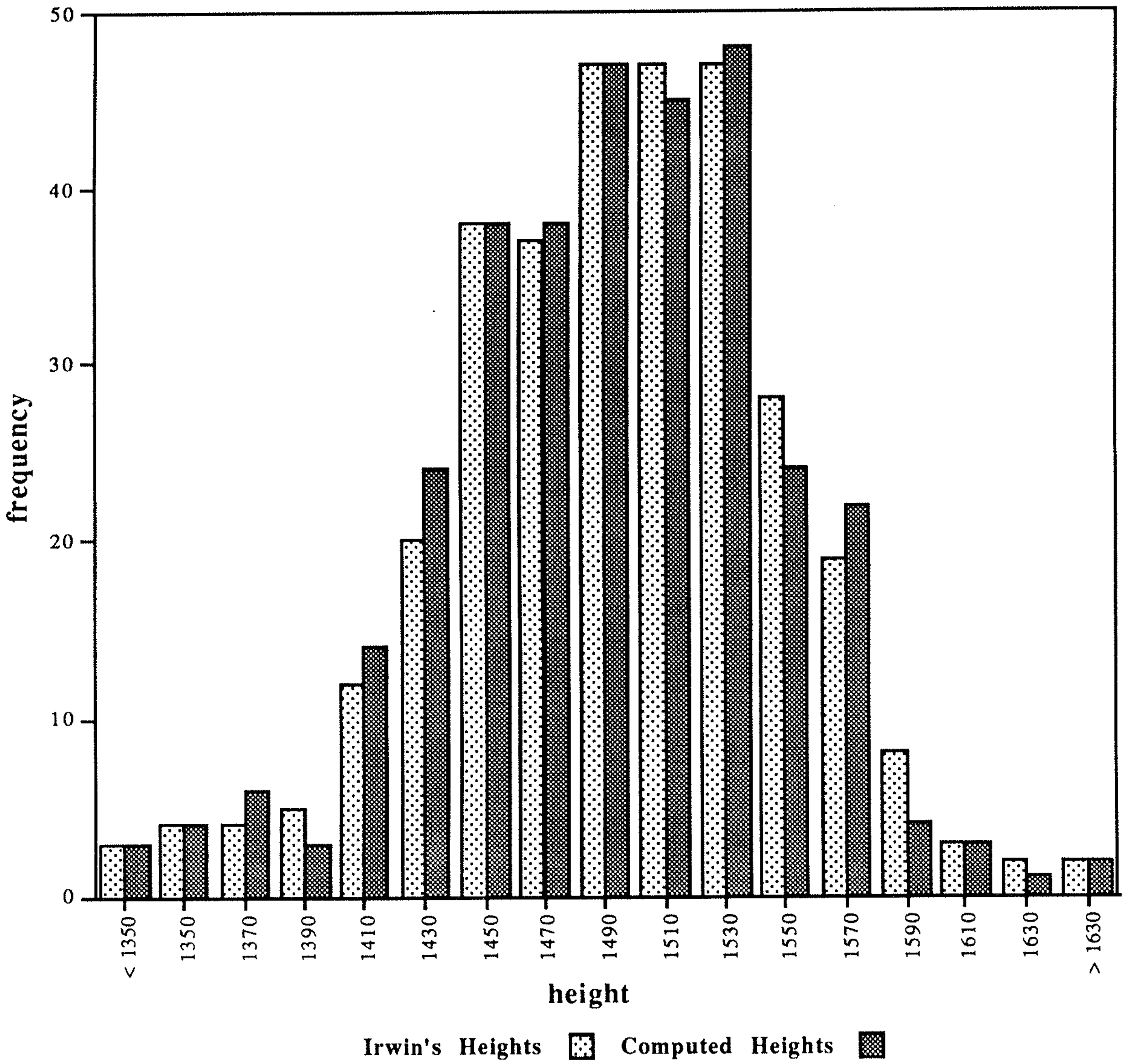


Figure 7

4 Syntheses

The final height for Mt. Sabine is computed from;

$$H_{ms} = H_i + h_c + h_o \qquad \text{(xii)}$$

where

H_{ms} = height of Mt Sabine

H_i = height difference as determined by Irwin,

h_c = height from Irwin's camp to the summit of Mt Sabine,

h_o = height of the base station at Cape Otway.

So

H_{ms} = 1494.29 + 200 + 300 = 1994.29 feet (607.86m)

This value can be compared to the present RL for Mt Sabine as, 583m, giving a difference of 24.86m. This value is within 1.44σ of Irwin's mean so it must be considered a good statistical result.

Acknowledgments

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VPRS780/7 Victorian Public Records Office, Vol. 780, Box 7.