Evolving Localised Learning for On-line Colour Image Quantisation

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Abstract

Although widely studied for many years, colour image quantisation remains a challenging problem. We propose to use an evolving self-organising map model for the on-line image quantisation tasks. Encouraging results are obtained in experiments and we look forward to implementing the algorithm in real world applications with further improvement.

1 Introduction

Colour image quantisation is a process for reducing the number of colours of a digital colour image. It is one of the most frequently used operations in computer graphics and image processing and is closely related to vector quantisation and image compression. Despite of the popularity of 24-bit graphics hardware, colour quantisation maintains its practical value. On the other hand, it remains a time consuming task although being widely studied for many years.

Given an image, typically a two-phase process is needed to reduce a true colour image into one with less colour resolution. The first phase, is to select the best representative colours into a colour map. The second, is to map each colour in the image to a corresponding colour in the colour map. There are two general classes of quantisation methods: fixed and adaptive. In fixed quantisation, a pre-defined set of display colours and a fixed mapping from the image colours to display colours are used. Fixed quantisation is very fast, but sacrifices the quantisation quality. In adaptive quantisation, colour space of the image is partitioned into clusters of a target number and the centroids of these clusters define the resulting colour map. Some popular implementations include median-cut [6], octree[7], and variance-based [11]. These methods mostly are based on colour histogram, requiring the whole set of image data to be obtained so that partitioning can proceed. They have much better quantisation results, but take much more time than fixed quantisation.

Another branch of the adaptive solution is connected with clustering algorithms [2][10][9]. So far they are not popularly applied yet, but since of their highly adaptive learning ability, they are promising as possible solutions to on-line image quantisation, for instance, progressive display of images and videos across the Internet.

In the context of data clustering and vector quantization (VQ), assume we have a data manifold χ of dimension D, i.e., $\chi \subseteq R^D$. We aim at finding a set of prototypes $\mathcal{W} = \{\mathbf{w}_1, ..., \mathbf{w}_N\}$, which encode the data manifold with small quantization error. VQ usually utilises a competitive rule, i.e., the input vector $\mathbf{x}$ is represented by the best matching unit $\mathbf{w}_{i(\mathbf{x})}$, which satisfies

$$\|\mathbf{x} - \mathbf{w}_i\| \leq \|\mathbf{x} - \mathbf{w}_j\|, \forall j \neq i, i, j \in [1, N] \tag{1}$$

The goal is to minimise the reconstruction error

$$E = \int d^D x P(\mathbf{x})(\mathbf{x} - \mathbf{w}_{i(\mathbf{x})})^2 \tag{2}$$

Here $P(\mathbf{x})$ is the probability distribution of data vectors over the manifold χ.

A straightforward approach for clustering and VQ is the well known K-means algorithm, whose on-line version is [8]:

$$\Delta \mathbf{w}_i = \begin{cases} \epsilon(\mathbf{x} - \mathbf{w}_i), & \text{if } i = i(\mathbf{x}) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

with ϵ as the step size. Such an on-line learning rule can suffer from confinement to local minima. A solution to this is to adopt some "soft" computing schemes in which not only the "winner" prototype is modified, but all reference vectors are adjusted depending on their proximity to the input vector.

Kohonen's self-organizing feature map (SOM) [5] is another VQ-related algorithm fallen in the neural network category. SOM has been used for vector quantisation of speech and image signals, featuring topology preserving ability and approximation of data distribution. The topology of the low dimensional feature map is pre-determined. This is good for visualisation purpose, but it also limits its data modelling ability, as the data manifold can be rather complicated. The size of SOM is also fixed and therefore it is not an ideal choice for on-line tasks.

The constraint of a low dimensional map topology is removed in the neural-gas model [8], with a learning rule similar to SOM, but the prototype vectors are organised in the original manifold of the input space. The weight updating rule is similar to that of SOM, but requiring the calculation of neighbourhood rank of the prototypes related to the current input, since no topology order is defined. This brings up the time complexity for each step of weight adapting. In [4] Fritzke introduced a growing neural gas (GNG) model which originates from the neural gas, but the network is allowed to adaptively grow. GNG needs to calculate local resources for prototypes, which introduces extra computational effort and reduces their efficiency.

We propose an evolving self-organising map (ESOM) [3], as an evolving extension of SOM for on-line VQ and classification tasks. It features fast one-pass incremental learning, evolvable network structure and good topology preserving ability. In this paper, we will further explore the plausibility of using this new algorithm to tackle the problem of colour image quantisation.

2 The ESOM algorithm

Following the context of vector quantisation and SOM, our approach is to allow the feature map to be evolved quickly and acquire topological representation in the same time. The neighbourhood of neurons are not pre-defined, but are dealt 'as-it-is', according to their present distances from each other. Thus ESOM avoids the time complexity of searching for neighbourhood ranking as in the neural gas algorithm.

Activations on prototype nodes are first defined. Given an input vector $\mathbf{x}$, the activation on the i-th node is defined as a matching score:

$$a_i = e^{-\|\mathbf{x}-\mathbf{w}_i\|^2/\epsilon^2} \tag{4}$$

where ϵ is a radial.

ESOM adopts a soft-winning competitive mechanism, in which the best winning unit and its neighbours are updated. We define a cost function as weighted quantisation error for the input vector:

$$E = \int d^D x \sum_{i=1}^{N} P(\mathbf{x}) g_i(\mathbf{x}) \|\mathbf{x} - \mathbf{w}_i\|^2 \tag{5}$$

Here $g_i(\mathbf{x})$ is a weighting factor assigned to the i-th prototype vector. Let $g_i(v) = a_i(v)$, i.e., the matching is weighted by the matching score itself, and the prototype which matches better should also contribute more to the matching error.

The on-line stochastic approximation of Eq.(5) gives:

$$E_{app} = \Sigma a_i \|\mathbf{x} - \mathbf{w}_i\|^2 \tag{6}$$

By gradient descent we have the following weight updating rule in a simplified form:

$$\Delta \mathbf{w}_i = \gamma a_i (\mathbf{x} - \mathbf{w}_i) \qquad i \in [1, N] \tag{7}$$

Here γ is a learning rate held as a small constant.

ESOM starts with a null network, and gradually allocates new prototypes when new data samples can not be matched well onto existing prototypes. The node insertion is straightforward. The new node is inserted representing exactly the poorly matched input vector. When handling clustered data, this simple approach shows advantage over the mid-point interpolation heuristics used in GNG. Although direct allocation is sensitive to noise and may introduce some artifacts in clustering, this can be mitigated by automatic deletion of obsolete nodes.

The ESOM algorithm is summarised in Fig.1.

3 Experimental results

We apply the ESOM algorithm in the problem of colour image quantisation and compare the results with those achieved by other methods including median-cut, octree, and Wu's method [12]. Three test images are chosen: Pool Balls, Mandrill and Lenna, as shown in Fig.2. The Pool Balls image is artificial and contains smooth colour tones and shades. The Mandrill image is of 262144 (512×512) pixels but has a very large number of colours (230427). The Lenna image is widely used image processing literature and contains both smooth areas and fine details. With all these images very good quantisation results are obtained with ESOM, shown in Fig.3.

1. Input a new data vector $\mathbf{x}$;
2. If there are no prototype nodes, go to **insertion**;
3. **Matching**. Look for a prototype subset $\mathcal{S}$, whose elements fall within the threshold distance from the input:

$$\mathcal{S} = \{\mathbf{w}_i \mid \|\mathbf{x} - \mathbf{w}_i\| < \epsilon, \mathbf{w}_i \in \mathcal{W}\}$$

   If $\mathcal{S}$ is *null*, go to **insertion**;
   Otherwise, calculate activations on nodes in $\mathcal{S}$ using Eq.(4), go to **updating**;
4. **Insertion**. Create a new node in the network representing the input:

$$\begin{aligned} \mathbf{w}_{N+1} &= \mathbf{x} \\ N &\leftarrow N+1 \end{aligned} \tag{8}$$

5. **Updating.** Modify all prototypes in $\mathcal{S}$ with Eq.(7).
6. **Deletion**. After every T_p steps of learning time, prune the weakest connection. If isolated nodes appear, prune them as well;
7. Go back to Step 1 (until no more data are available).

Figure 1: The ESOM algorithm summarised.

(a) (b) (c)

Figure 2: The source images. (a) Pool Balls, (b) Mandrill. (c) Lenna.

The RGB colour space is used directly in the clustering. Here we denote the image as I, with a pixel number of N. The input vector to the ESOM algorithm is now a 3-dimensional one: $I_i = (R_i, G_i, B_i)$. The on-line clustering process of ESOM will construct a colour map $\mathcal{C} = \{c_j | j = 1...256\}$. Each image pixel is then quantised to the best-matching palette colour c_m, a process denoted as

$$Q : I_i \rightarrow c_m$$

To speed up the calculation process, the L_α norm [1] is adopted as an approximation of the Euclidean metric used in ESOM. It is defined as

$$\|\mathbf{x}\|_\alpha = (1-\alpha)\sum_{j=1}^{n} |x_j| + \alpha \mathtt{max}_{j=1}^{n} |x_j| \tag{9}$$

where the vector $\mathbf{x} \in R^n$. We use $\alpha = 1/2$.

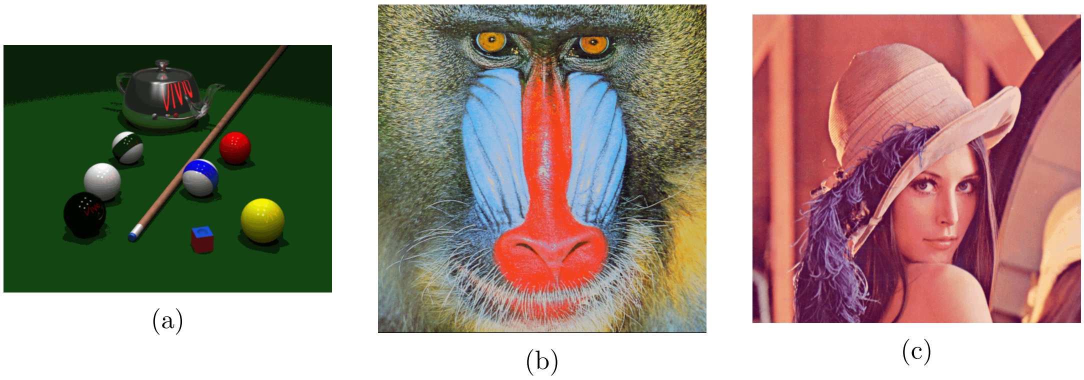

Figure 3: Test images quantised to 256 colours. (a) Pool Balls, (b) Mandrill. (c) Lenna.

The quantisation root mean square error (QRMSE) is defined as

$$\epsilon_{I,Q} = \left[\frac{1}{N} \sum_{i=1}^{N} d(I_i, c_m)^2 \right]^{\frac{1}{2}} \tag{10}$$

Apart from the quantisation error, quantisation error variance is another factor which influences the visual quality of the quantised image. The standard deviation of error is defined as

$$\sigma = \left[\frac{\sum_i (\|I_i - c_m\| - \epsilon_{I,Q})^2}{N} \right]^{\frac{1}{2}} \tag{11}$$

Quantisation performance of different methods are compared in Table 1.

Table 1: Quantisation performances: quantisation error / error deviation

Methods	Pool Balls	Mandrill	Lenna
Median-cut	2.58 / 8.28	11.32 / 5.59	6.03 / 3.50
Octree	4.15 / 3.55	13.17 / 4.98	7.56 / 3.83
Wu's	2.22 / 2.19	9.89 / 4.56	5.52 / 2.94
ESOM	6.06 / 4.73	11.85 / 5.90	6.58 / 2.91

Our experiments show that while ESOM produces small quantisation error and error deviation measures, the visual quality of quantised images is better than both median-cut and octree methods and is comparable with Wu's method, which gives the best performance.

As for the speed of the algorithms, ESOM does not tell a happy story yet. With the 512×480 sized Lenna image, it takes 2 seconds to construct the quantisation palette on a Pentium-II system running Linux 2.2. We believe, however, there are a number of ways to reduce the time consumed both in palette updating and in colour matching, for example, building parallelised implementation, or organising the palette into tree structures.

4 Conclusion

We propose to use an evolving localised learning model for on-line colour image quantisation. Promising results have been obtained with some benchmark colour images. With further improvement on the computation efficiency, we will investigate in applying our algorithm to progressive image display and video display.

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